LEVERAGED SMALL VALUE EQUITIES

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Abstract

The size premium and value premium are well documented in academic studies. We contribute to this literature by finding that leverage – as defined by long term debt divided by enterprise value – enhances the average returns of a small-value investment strategy. At the company level, our results indicate that there is a positive interaction between leverage and value. We test a variety of quality and technical factors to develop a theory of what works in leveraged small-value equity investing. We develop a ranking system for creating annual portfolios of leveraged small-value stocks in the United States. This ranking system prioritizes smaller, cheaper and more leveraged stocks that are already paying down debt and exhibit improving asset turnover. Annual portfolios of the top 25 stocks in this ranking system have a 25.1% average annual return between 1965 and 2013. At a standard deviation of 39.4%, the Sharpe Ratio of these annual portfolio returns is 0.51. These portfolios have a CAPM alpha of 9.6% and a CAPM beta of 1.66. The average risk-adjusted return of these portfolios is 13.1% per year after controlling for the 3 Fama-French factors, momentum, and liquidity. In addition to providing a novel investment strategy, we believe that our findings have implications for the leveraged buyout industry—which follows an analogous investment approach in the private markets—as well as for public market value investors who have traditionally eschewed leverage.

1. Introduction

Financial leverage, as measured by long-term debt divided by enterprise value (EV), improves the returns of a small-value investment strategy. Practitioners in the leveraged-buyout industry have widely understood the key elements of this "small-value on steroids" strategy, adding leverage to small-value companies in the private markets to great success.[1] Our paper tests these variables in the public equity markets and provides an in-depth quantitative exploration of how to successfully invest in leveraged small-value stocks in the public equity markets.

Our study is based on an analysis of U.S. stocks between January 1964 and December 2014, which were compiled from the Center for Research in Security Prices (CRSP) database. Specifically, we analyze the returns of annual portfolios that are drawn from a target universe of stocks that are cheap (below 25^{th} percentile of *EBITDA/EV*), small (25^{th} to 75^{th} percentile of market capitalization) and leveraged—with above median *Long Term Debt/EV* relative to all NYSE/AMEX/NASDAQ stocks in a given year.

We find that that there are five factors that are statistically significant in predicting returns within this universe. In order of significance, those factors are: *debt pay-down* $(LT\ Debt_{YEAR\ T-1} < LT\ Debt_{YEAR\ T-2})$, *LT Debt/EV*, improving *asset turnover* $(\%\ Revenue\ Growth > \%\ Asset\ Growth)$, *market capitalization*, and *EBITDA/EV*. We provide a theoretical explanation for this observed behavior, focused on the idea that the excess returns from this strategy come primarily through deleveraging, a virtuous cycle of reduced interest payments, improved financial stability, and value accrual for equity investors. Free cash flow yields and

[1] We would like to thank Professor Robert Vishny for providing the theoretical framework of private equity returns as "small-value on steroids" in his Behavioral and Institutional Finance class at Chicago Booth.

business quality metrics help best identify the companies that are most capable of paying back their debt.

We also find some evidence for short term reversals in one-year price performance; stocks that had below-median returns in the prior year tend to outperform the above-median-return stocks from that period in the subsequent year. This relationship is stronger among the below-median-return stocks that had low share turnover in the prior year. Our observation of short term reversals among low share turnover, "past loser" stocks is consistent with the research published by Lee and Swaminathan (2000), who find that firms with low (high) past turnover ratios exhibit many value (glamour) characteristics and earn higher (lower) future returns.[2]

We develop a ranking system based on these factors that produces a robust and novel investment strategy. This ranking system distinguishes companies that generate attractive returns from those that underperform. The *Top 25 Portfolios'* annual equal-weighted returns exceed the market by 11.7 percentage points on average. The annual equal-weighted returns of the *Top 50 Portfolios* exceed the market by 9.2 percentage points on average. Similarly, the *Q1 Portfolios'* annual equal-weighted returns exceed the market by 9.1 percentage points on average. All three of these results are statistically significant at t=4.65, t=4.90, and t=5.65 respectively. There is no evidence that the *Q2 Portfolios* or the *Q3 Portfolios* generate average returns that are different from the market. The *Q4 Portfolios* underperform the universe by a statistically significant 3.6 percentage points per year on average. Our ranking system is robust in terms of identifying winners and losers in the universe of leveraged small-value stocks. The ranking system assigns better ranks to stocks that have higher expected returns, and it assigns worse ranks to the least attractive stocks that have lower expected returns.

[2] "Price Momentum and Trading Volume" Charles M. C. Lee and Bhaskaran Swaminathan (2000)

The rest of this paper is organized as follows: Section 2 summarizes our data, Section 3 outlines the parameters we used to form a universe of leveraged small-value stocks, and Section 4 presents an analysis of that universe. Section 5 discusses investment perspectives and Section 6 concludes with a discussion of leverage aversion and the volatility that is inherent in our strategy.

2. Data

Our analysis uses data on NYSE, AMEX, and NASDAQ stocks from the Center for Research in Securities Prices (CRSP). We use annual accounting information for these stocks from January 1, 1963 to December 31, 2012. We utilize quarterly price and dividend information between January 1, 1964 and December 31, 2014 in order to calculate portfolios returns. The portfolios are formed at the end of the first calendar quarter in each year, based on accounting information from the previous calendar year, in order to prevent look-ahead bias. For example, the portfolio formed on March 31, 1986 uses annual accounting information from the 1985 calendar year. We use current prices in all cases so the return of the 1986 portfolio is a function of prices at the end of 1Q 1986 and prices at the end of 1Q 1987, including dividends that are received between these dates. Our analysis includes 49 portfolios, with the first being formed at the end of 1Q 1965 and the last being formed at the end of 1Q 2013. Each portfolio is held for one year and we use value-weighted portfolio returns for the benchmarking and factor analyses in Section 5.

In order to account for missing price information, we used CRSP's delisting dataset to recover final prices for stocks that delisted from the data. In cases where price information was still missing after reconciling against CRSP's delisting dataset, we assumed a -100% return. This assumption applied to less than 1% of the firms in our data. Overall, the equal-weighted average annual return for all stocks in our dataset is 15.6% between 1964 and 2014. This result is similar to the 16.5% equal-weighted annual return that CRSP reflects in its NYSE/AMEX/NASDAQ index between 1964 and 2014.

3. Universe

Each year, we sorted stocks independently according to size, value and leverage. Our analysis focuses on the universe of stocks that meet all three of the criteria listed below.

1. **Companies in the 25th to 75th percentile of market capitalization each year.** This rule is designed to capture the size premium while excluding micro-cap stocks that may be too small for institutional investors. In all cases, market capitalization was defined as the number of common shares outstanding at the end of the calendar year preceding portfolio formation (i.e. on December 31 of Year T-1), multiplied by the stock price on the date of portfolio formation (i.e. on March 31 of Year T).

   The share prices in CRSP's database are all adjusted for stock splits. Therefore, the market capitalizations in 2013 are better reflections of today's market values than older vintage years. A summary of the market capitalizations of stocks that fell between the 25th and 75th percentile in 2013 is provided below.

   Figure 1 – Market Cap Distribution of Small Stocks in 2013

   $ in millions

Variable	Obs	Mean	Median	Std. Dev.	Min	Max
Market Cap	2,574	$715.38	$500.66	$611.07	$83.94	$2,380.20

2. **25% cheapest companies in each year based on *EBITDA/EV*.** This rule aims to capture the value premium. The *EBITDA/EV* metric can plausibly provide a more current measure of valuation than *Price/Book* because book value is generally a stale accounting measure, whereas enterprise value and EBITDA both incorporate the most recently

available fundamental information. In the case of this analysis, enterprise value was defined as the sum of market capitalization and long term debt.

3. **Above-median leverage ratio relative to the universe of U.S. stocks in each year.** The leverage ratio is defined as *LT Debt/EV*. As such, the leverage ratio is bounded between 0 and 1. We used above-median leverage as the selection criteria—as opposed to top quartile leverage—in order to allow greater scope for debt pay-down.

In the table below, we demonstrate the equal-weighted average annual returns that are associated with value and leverage among small-cap stocks in regression 1 and 2. In regression 3, we show the performance of the universe of leveraged small-value stocks relative to the market. In all cases, the dependent variable is the one-year return of each stock from Year T_0 to Year T_0+1. The independent variables *Value* and *Leveraged* are both binary (0 or 1) variables. *Value* is equal to 1 if a stock is among the cheapest 25% of stocks by *EBITDA/EV* in its year. *Leveraged* is equal to 1 if a stock's *LT Debt/EV* is above the median threshold for its year. In regression 1, we restrict our analysis to small stocks (25th to 75th percentile by market cap) and we regress *Next 1 Year Return* against the *Value* and *Leveraged* binary variables. In regression 2, we conduct the same analysis but with time fixed-effects in order to control for market conditions that vary by year. Based on regression 2, we see that among small equities, value stocks outperform their peers by 5 percentage points per year, on average. This result is statistically significant (t=8.70). Similarly, there is a return premium of 4 percentage points per year that is associated with being a leveraged stock. Finally, in regression 3, we use the full database of stocks and regress *Next 1 Year Return* against a binary *Universe* variable that is equal to 1 for stocks that meet all three criteria in terms of size, value and leverage. Regression 4 repeats this analysis with time fixed-effects.

Figure 2 – Analysis of Leveraged Small Value Universe

Regression:	(1)	(2)	(3)	(4)
Scope of Analysis:	*Small Stocks*	*Small Stocks with Time F.E.*	*All Stocks*	*All Stocks with Time F.E.*
Dependent Variable:	Next 1 Year Return			
Value	0.0559	0.0502		
	(9.19)	(8.70)		
Leveraged	0.0419	0.0405		
	(6.59)	(6.63)		
Universe			0.0141	0.0188
			(1.87)	(2.66)
Intercept	0.0754	Time Fixed Effects	0.1553	Time Fixed Effects
	(18.18)		(40.23)	
R^2	0.0012	0.0833	0.0000	0.0280
Number of obs	105,708	105,708	232,130	232,130

As shown in regression 4, the universe of leveraged small value stocks outperforms the market by a statistically significant 1.88 percentage points per year. Therefore, there is evidence that this universe is an attractive fishing pool for a value investor. It is also a deep pool, with 343 stocks in 2013. Figure 3 provides summary statistics for this universe over the entire 49 year horizon in our analysis between 1965 and 2013.

Figure 3 – Summary Statistics for the Leveraged Small Value Universe: 1965-2013

	EBITDA/EV	LT Debt/EV	Gross Profit/Assets	LT Debt/Assets
Mean	22.7%	38.8%	33.2%	25.8%
Standard Deviation	14.6%	18.7%	24.5%	17.0%
10th Percentile	14.1%	16.7%	5.6%	5.4%
25th Percentile	16.5%	23.7%	16.2%	14.8%
Median	20.0%	35.7%	28.9%	23.7%
75th Percentile	25.6%	50.9%	43.7%	34.0%
90th Percentile	33.5%	65.6%	61.9%	46.7%
Number of Stocks	15,607	15,607	15,607	15,607

4. Analysis

Within the universe of leveraged small-value stocks, we set out to identify the factors that separate the attractive investments from the less attractive investments. We categorize these factors as deleveraging factors, technical factors, and quality factors.

A. Deleveraging Factors

Free cash flow drives deleveraging. The best metric for deleveraging potential is a company's free cash flow yield (*free cash-flow/market capitalization*). A company's free cash flow yield varies mechanistically relative to valuation as a multiple of free cash flow and the amount of leverage in the capital structure.

In a world with no debt, there is only one way to increase the free cash flow yield, which is to pay a lower price for the business. Paying double the price means getting half the free cash flow yield. But in a world in which companies can borrow, they can also use debt to enhance their free cash flow yield. Taking on debt reduces the amount of equity needed to own the full business (thereby reducing the denominator in the free cash flow yield equation). But debt also has a second impact; adding interest payments that reduce free cash flow (the numerator).

This second effect is key to understanding why valuation matters so much more in leveraged equities than in the broader markets. Holding leverage fixed as a percent of purchase price, any increase in the purchase price has a doubly negative impact on free cash flow yield: first by increasing the interest load and second by increasing the absolute size of the equity account.

Leverage therefore magnifies the impact of valuation on the free cash flow yield of a business. In an unlevered equity, free cash flow yield changes only gradually as the purchase price increases.

However, as you increase the amount of leverage, the difference in free cash flow yield is amplified. Figure 4 below illustrates this point through a sensitivity analysis.

Figure 4 – Free Cash Flow Yield Varying by EBITDA Multiple and Leverage

		Net Debt as % of Total Enterprise Value					
		0% ...	**50%**	**60%**	**70%**	**80%**	**90%**
	3x	16% ...	27%	33%	42%	60%	116%
	5x	10% ...	14%	17%	20%	28%	51%
	7x	7% ...	9%	10%	11%	14%	23%
EBITDA	**9x**	5% ...	6%	6%	6%	6%	7%
Entry	**11x**	4% ...	4%	3%	3%	1%	-2%
Multiple	**13x**	4% ...	2%	2%	0%	-2%	-9%
	15x	3% ...	1%	0%	-1%	-5%	-14%
	17x	3% ...	1%	-1%	-3%	-6%	-18%
	19x	3% ...	0%	-1%	-4%	-8%	-21%

In the far left column of Figure 4, which represents an unlevered equity, free cash flow yield changes only gradually as the purchase price increases. The difference in free cash flow yield between paying 5x EBITDA and 15x EBITDA is only 7 percentage points. However, as leverage increases, the difference in free cash flow yield is larger. At 70% leverage, paying 5x EBITDA results in a 20% free cash flow yield, while paying 15x results in a negative 1% free cash flow yield; a 21 percentage point difference in free cash flow yield. This sensitivity provides a model for the interaction between leverage and valuation. We include an interaction variable in our regressions to test this effect. The interaction variable has a positive coefficient although it is not statistically significant at the 10% level in a time fixed-effects regression.

However, these two variables both only roughly bound free cash flow yield – EBITDA is an imperfect proxy for actual cash flow. Interest rates vary, tax rates vary, and capital intensity varies. So we balance the use of the cleaner and less noisy *EBITDA/EV* and *LT Debt/EV* variables with the most direct measure of free cash flow generation – debt paydown. Since this variable can be very noisy, much more variable than EBITDA for example, we measure this by assigning a binary 1 or 0 variable according to whether a company has reduced long term debt in the previous year, as measured by $(LT\ Debt_{YEAR\ T-1} < LT\ Debt_{YEAR\ T-2})$.[3] This is the most statistically significant variable in our data set. Cheap and leveraged companies that have been actively paying down debt are in a different position from companies that have recently raised debt because of an inability to generate sufficient internal funds. Using cash flow to deleverage has a number of positive impacts: reducing the risk of financial distress, reducing interest payments, and increasing equity value.

B. Technical Factors

In addition to deleveraging factors which are important from a fundamental business perspective, we also tested technical variables. These variables are size ($\ln(Market\ Cap)$), share turnover $\left(\ln \frac{Number\ of\ Shares\ Traded_{YEAR\ T-1}}{Average\ Number\ of\ Shares\ Outstanding_{YEAR\ T-1}}\right)$ and prior-year return (*PY Return Below Median*). The size and share turnover variables are expressed in logarithmic form because of their wide range in values. The *PY Return Below Median* variable is binary and it is equal to 1 if a stock's prior-year return was below the median threshold for all stocks in that preceding year. The above-median prior-year return stocks have a value of 0 in this variable. Of these factors, only size is statistically significant at the 10% level in a time fixed-effects regression.

C. Quality Factors

Publicly-traded companies that show high levels of leverage and low valuations are typically facing some risky or problematic situation, declining revenues or profitability, or other unfavorable circumstances. The companies' historical financial statements can provide clues that

[3] "Value Investing: The Use of Historical Financial Statement Information to Separate Winners from Losers": Joseph D. Piotroski (Jan 2002)

help to separate the businesses with stable or improving financial conditions from those that are at higher risk of financial distress.

Improving asset turnover is the most statistically significant quality factor. We use a binary variable to identify companies with improving asset turnover in the past year ($\%\ Revenue\ Growth > \%\ Asset\ Growth$). Improvements in asset turnover can either come from more efficient operations (fewer assets generating the same amount of sales) or an increase in sales (which could signify improved market conditions for a company's products).[4] Deterioration in asset turnover can be a negative predictor; signaling a declining end market for the company's products or inefficient capital spending. With respect to the latter, Titman, Wei, and Xie (2004) find that firms with substantial increases in capital investment earn negative benchmark-adjusted returns.[5] As such, asset growth may also be related to the conservative-minus-aggressive (CMA) factor in Fama and French's five factor model, whereby firms that invest conservatively earn higher average returns than firms that invest aggressively.[6]

Looking at profitability and leverage relative to book assets can also provide additional clues to help separate the attractive leveraged small-value stocks from the unattractive leveraged small-value stocks. Novy-Marx (2012) shows that value strategies can be improved by controlling for gross profitability (*Gross Profit/Assets*). All else equal, investors should prefer companies that produce higher gross profits relative to assets since more productive assets are clearly preferable to less productive assets.[7] Investors should also prefer companies that have low or moderate levels of leverage relative to assets. A company that has borrowed an amount equivalent to half

[4] "Value Investing: The Use of Historical Financial Statement Information to Separate Winners from Losers": Joseph D. Piotroski (2002)
[5] "Capital Investments and Stock Returns" Sheridan Titman, K.C. John Wei, Feixu Xie (2004)
[6] "A Five-Factor Asset Pricing Model" Eugene F. Fama and Kenneth R. French (2014)
[7] "The Other Side of Value: The Gross Profitability Premium " Robert Novy-Marx (2012)

the value of its assets is in a stronger and more stable financial position than a company that has borrowed an amount equivalent to twice its asset base.

Preferring companies that are moderately leveraged relative to assets might seem contradictory to a preference for firms that are more leveraged relative to enterprise value. However, these preferences are in fact complementary for a value investor. Another way to think about this relationship is that conditional on two companies being identical in terms their fundamentals, including *LT Debt/Assets* and *Gross Profit/Assets*, the firm that has a higher *LT Debt/EV* must have a lower valuation and it should be more attractive to a value investor. For example, imagine two identical companies that each produce the same level of gross profit relative to assets and have the same leverage relative to assets. The only way for one company to have a higher level of leverage relative to enterprise value is for that company's enterprise value to be lower.

Therefore, the ideal leveraged stock has high *Gross Profit/Assets*, and low-to-moderate *LT Debt/Assets*, a low valuation in terms of *EV/EBITDA*, and a high level of *LT Debt/EV*.

D. Regression Results

Figure 5 presents the results of our analysis. In regression 5 and regression 6a, we focus on the universe of leveraged small-value stocks. Regression 5 evaluates the average effect of our factors on *Next 1 Year Return* (among universe stocks) and controls for time using a trend by portfolio year. Regression 6a repeats this analysis but controls for time using time fixed-effects. In regression 6b, we apply the time-fixed effects model on all stocks in our database as a robustness test for the factors in the aforementioned regressions. We believe that regression 6a is the best model for explaining the variation in returns within our universe of leveraged small-value stocks. This regression also has a higher R-squared at 19.6%.

Figure 5 – Regressions of One-Year Returns on Factors

In equation 5, we regress the *Next 1 Year Return* of universe stocks on the factors in our model and we control for time using a trend by portfolio year. Regression 6a conducts the same analysis on universe stocks, but controls for time using time fixed-effects. Regression 6b is a generalized version of regression 6a, and it applies the same time fixed-effects regression on *all* stocks in our database. The t-statistic of each coefficient is shown in parentheses.

Regression:	**(5)**	**(6a)**	**(6b)**
Scope of Analysis:	***Universe Stocks***	***Universe Stocks with Time F.E.***	***All Stocks with Time F.E.***
Dependent Variable:		**Next 1 Year Return**	
Debt Paydown	0.0359	0.0362	0.0223
	(2.63)	(2.84)	(2.95)
ln(LT Debt/EV)	0.4527	0.1891	0.0215
	(4.91)	(2.21)	(3.34)
Asset Turnover	0.0370	0.0265	0.0409
	(2.87)	(2.20)	(6.09)
ln(Market Cap)	-0.0366	-0.0152	-0.0410
	-(4.56)	-(1.91)	-(15.14)
ln(EBITDA/EV)	0.4916	0.1166	0.0630
	(7.32)	(1.90)	(5.62)
ln(Share Turnover)	-0.0089	-0.0146	-0.0177
	-(0.91)	-(1.48)	-(4.42)
PY Return Below Median	0.0129	0.0162	0.0572
	(1.10)	(1.46)	(9.46)
ln(EBITDA/EV) x ln(LT Debt/EV)	0.1702	0.0677	0.0079
	(3.24)	(1.37)	(2.85)
Gross Profit/Assets	0.0769	0.0310	0.0767
	(2.70)	(1.18)	(4.62)
LT Debt/Assets	-0.1532	-0.0505	0.1649
	-(2.35)	-(0.84)	(3.69)
Share Repurchases	-0.0020	0.0007	0.0120
	-(0.13)	(0.05)	(1.72)
Portfolio Year	0.0120		
	(12.70)		
R^2	0.0460	0.1964	0.0484
Number of obs	14,511	14,511	139,448

Regression 6a highlights the importance of debt pay-down in a leveraged small-value strategy. All else equal, companies in this universe that are already paying down debt earn an average annual return that is 3.6 percentage points higher than firms that have not reduced long term debt. At (t=2.84), this result is the most statistically significant single effect in regression 6a.

5. Investment Perspective

Based on the results of regression 6a, we developed a ranking system for creating annual portfolios. Based on the ranking results for each year, we formed annual portfolios between 1965 and 2013. These portfolios comprise (i) the 25 highest ranking stocks in each year (*Top 25 Portfolios*), (ii) the 50 highest ranking stocks in each year (*Top 50 Portfolios*), and (iii) four portfolios that group stocks according to their quartile ranking (*Q1 Portfolios*, *Q2 Portfolios*, *Q3 Portfolios* and *Q4 Portfolios*). Using binary variables that are equal to 1 for stocks assigned to the relevant portfolios, we regressed the returns of these portfolios against the full database of NYSE/AMEX/NASDAQ stocks. These regressions provide a test of the ranking system by measuring the portfolios' performance relative to the market. The results are summarized in Figure 7.

Based on these results, our ranking system seems to be an effective way to identify companies that generate attractive returns. The *Top 25 Portfolios'* equal-weighted annual returns exceed the market by 11.7 percentage points on average. The equal-weighted annual returns of the *Top 50 Portfolios* exceed the market by 9.2 percentage points on average. Similarly, the *Q1 Portfolios'* equal-weighted annual returns exceed the market by 9.1 percentage points on average. All three of these results are statistically significant at t=4.65, t=4.90, and t=5.65 respectively. There is no evidence that the *Q2 Portfolios* or the *Q3 Portfolios* generate average returns that are different from the market. The *Q4 Portfolios* underperform the market by a statistically significant 3.6 percentage points per year on average. As such, the regressions in Figure 7 provide evidence that our ranking system is robust in terms of identifying winners and losers in the universe of leveraged small-value stocks. The ranking system assigns better ranks to stocks that have higher expected returns, and it assigns worse ranks to the least attractive stocks that have lower expected returns. Finally, we present the average annual returns of each portfolio category in Figure 8, along with the respective standard deviations and Sharpe Ratios. Figure 8 also presents value-weighted returns for each portfolio, which we use for all benchmarking and performance evaluation analyses henceforth. As was the case in Figure 7, the returns in Figure 8 represent all portfolios that were formed between 1965 and 2013.

Figure 7 – Regressions of Next 1 Year Returns against Portfolio Assignments

In each of the regressions below, we regressed the *Next 1 Year Return* against a binary variable that indicates a universe stock's portfolio assignment. The regressions capture all NYSE/AMEX/NASDAQ stocks in our database. The formula for each regression is as follows:

$$Next\ 1\ Year\ Return = \alpha + \beta * Portfolio + \varepsilon$$

The coefficients for each portfolio assignment represent the average annual excess return that each portfolio achieves relative to the market in equal-weighted terms. The intercept represents the average equal-weighted market return. T-statistics are provided in parentheses below each coefficient as a measure of statistical significance.

Regression:	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**	**(12)**
Scope of Analysis:	***All Stocks***					
Dependent Variable:	**Next 1 Year Return**					
Binary Independent Variable:	**Top 25 Portfolios**	**Top 50 Portfolios**	**Q1 Portfolios**	**Q2 Portfolios**	**Q3 Portfolios**	**Q4 Portfolios**
Coefficient	0.1172	0.0916	0.0906	-0.0010	-0.0006	-0.0357
	(4.65)	(4.90)	(5.65)	-(0.07)	-(0.05)	-(3.59)
Intercept	0.1555	0.1552	0.1547	0.1561	0.1561	0.1567
	(42.77)	(42.49)	(42.17)	(42.54)	(42.52)	(42.64)
R^2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Number of obs	232,694	232,694	232,694	232,694	232,694	232,694

Figure 8 – Summary of Portfolio Returns

In the tables below, we summarize the average annual returns, standard deviation, and Sharpe Ratio of each set of portfolios. The top table presents equal-weighted portfolio returns and the bottom table presents value-weighted portfolio returns.

Equal-Weighted	Top 25 Portfolios	Top 50 Portfolios	Q1 Portfolios	Q2 Portfolios	Q3 Portfolios	Q4 Portfolios
Average Annual Return	27.3%	24.7%	23.3%	15.4%	14.4%	11.3%
Standard Deviation	41.8%	42.2%	40.4%	35.8%	29.4%	27.1%
Sharpe Ratio	0.53	0.46	0.45	0.29	0.32	0.23

Value-Weighted	Top 25 Portfolios	Top 50 Portfolios	Q1 Portfolios	Q2 Portfolios	Q3 Portfolios	Q4 Portfolios
Average Annual Return	25.1%	23.0%	22.0%	16.7%	15.7%	11.6%
Standard Deviation	39.4%	40.0%	41.9%	34.2%	29.5%	27.1%
Sharpe Ratio	0.51	0.45	0.40	0.34	0.36	0.24

Having established that an attractive investment strategy can be developed from selecting stocks at the top of our ranking system, we then turned to the issue of benchmarking. The primary portfolios of interest from an investment perspective are the *Top 25 Portfolios*. We also evaluated the *Top 50 Portfolios* and the *Q1 Portfolios*. In order to identify the most appropriate benchmark for our leveraged small-value strategy, we regressed the value-weighted excess returns of each portfolio above the risk free rate ($r_{PORTFOLIO} - r_{30\ DAY\ T-BILL}$) against the excess returns of the benchmarks ($r_{BENCHMARK} - r_{30\ DAY\ T-BILL}$). The objective of this exercise is to identify the benchmark that has the highest R^2 and therefore explains the largest amount of variation in our portfolios' returns. The results of this analysis are summarized in Figure 9. We used value-weighted portfolio returns and value-weighted benchmarks in all cases. It should be noted that the Cambridge Associate Private Equity Index is net of fund fees, unlike the other benchmarks and our portfolios, which do not incorporate any fees.

Based on the results in Figure 9, the most appropriate benchmarks for our leveraged small-value strategy are the Fama-French Small Value Index and the CRSP Small Value Index. Unsurprisingly, the Fama-French Index provides a higher R^2 over the full 1965-2013 period than the S&P 500. However, the Fama-French Index is not traded. On the other hand, the CRSP Small Value Index is traded and it is followed by the Vanguard Small-Cap Value Index Fund.[8]

Although data is only available for the CRSP Small Value Index starting in 2002, it works just as well as the Fama-French Index in explaining the variation in returns of our portfolios. A direct comparison of the Fama-French Index and the CRSP Small Value Index between 2002 and 2013 is provided in regression 13b and regression 14 in Figure 9.

[8] In April 2013, Vanguard transitioned its target benchmarks for U.S. funds from MCSI to CRSP Indexes.

Figure 9 – Benchmark Analysis

Regression:	(13a)	(13b)	(14)	(15)	(16)	(17)
Dependent Variable:	**Portfolio Excess Return Above 30-Day T-Bill (R_P - R_F)**					
Benchmark (R_B - R_f):	**Fama-French Small Value**	**Fama-French Small Value**	**CRSP Small Value**	**S&P 500**	**Russell 2000**	**CA Private Equity***
Time Horizon:	**1965 - 2013**	**2002 -2013**	**2002 -2013**	**1965 - 2013**	**1995 - 2013**	**1987 - 2013**
Top 25 Portfolios (β)	1.33	1.53	1.83	1.56	1.90	1.27
	(12.08)	(8.93)	(9.05)	(6.71)	(4.04)	(1.95)
Intercept	0.97%	21.41%	20.34%	10.15%	18.55%	12.37%
	(0.29)	(2.99)	(2.86)	(2.30)	(1.82)	(1.07)
R^2	**75.63%**	**88.85%**	**89.12%**	**48.95%**	**48.99%**	**13.15%**
Number of obs	49	12	12	49	19	27
Top 50 Portfolios (β)	1.39	1.65	1.96	1.57	1.82	1.11
	(14.10)	(9.41)	(9.30)	(6.71)	(3.55)	(1.65)
Intercept	-1.88%	10.45%	9.37%	8.02%	14.98%	10.79%
	-(0.64)	(1.43)	(1.26)	(1.80)	(1.35)	(0.90)
R^2	**80.87%**	**89.85%**	**89.64%**	**48.90%**	**42.62%**	**9.82%**
Number of obs	49	12	12	49	19	27
Q1 Portfolios (β)	1.41	1.74	2.07	1.53	1.79	0.84
	(12.35)	(7.75)	(7.56)	(5.93)	(3.03)	(1.15)
Intercept	-3.14%	5.76%	4.69%	7.26%	9.52%	11.53%
	-(0.93)	(0.62)	(0.49)	(1.48)	(0.74)	(0.89)
R^2	**76.44%**	**85.72%**	**85.09%**	**42.83%**	**35.01%**	**5.06%**
Number of obs	49	12	12	49	19	27

** Cambridge Associate's private equity benchmark is net of fees, which makes it less comparable to the other benchmarks*

In order to estimate the risk-adjusted returns (alpha) of the leveraged small-value portfolios, we started by estimating the CAPM alphas through a regression of the value-weighted portfolios' excess returns against the excess return on the market $(r_{MKT} - r_f)$. The results of this analysis are presented in regressions 18 – 20 in Figure 10. The *Top 25 Portfolios* have an annual CAPM alpha of 9.6% that is statistically significant at the 5% level. The *Top 50 Portfolios* have an annual CAPM alpha of 7.5% that is statistically significant at the 10% level. Finally, the *Q1 Portfolios* have an annual CAPM alpha of 6.7%, although this result is not statistically significant.

We then conducted factor regressions in each set of portfolios. We used the three Fama-French factors: excess return on the market (MKT), small-minus-big (SMB), and high-minus-low (HML). SMB and HML capture the size and value premium, respectively. We also included the up-minus-down momentum factor (MOM). In addition, we included Pástor and Stambaugh's traded liquidity factor (LIQ). Stocks that have a high beta to LIQ have higher exposure to liquidity risk in the market. These tend to be the smallest and most thinly traded stocks.[9] Pástor and Stambaugh (2002) find that stocks that have higher liquidity betas tend to earn higher average returns. Just like SMB, HML and MOM, the traded LIQ factor represents a long-short spread. Specifically, LIQ goes long the 10th decile of stocks with the highest predicted liquidity betas and it shorts the 1st decile of stocks that have the lowest liquidity betas. The alphas from the regressions that use all five factors represent our strategy's average annual risk-adjusted returns. The results of this factor analysis are presented in regressions 21 – 23 in Figure 10, using value-weighted portfolio returns.

[9] "Liquidity Risk and Expected Stock Returns" Luboš Pástor and Robert F. Stambaugh (2002)

Figure 10 – CAPM Alphas and Factor Analysis

In the tables below, we start by regressing the excess returns of each value-weighted portfolio against the market's excess returns in order to estimate the CAPM alpha of each set of portfolios. We then conduct a factor analysis using the 3 Fama-French factors, as well as momentum and the traded Pástor-Stambaugh liquidity factor. We use the following regression models in the tables below:

Regression 18 – 20: $r_{PORTFOLIO} - r_{30\ DAY\ T-BILL} = \alpha + \beta_1 MKT + \varepsilon$

Regression 21 – 23: $r_{PORTFOLIO} - r_{30\ DAY\ T-BILL} = \alpha + \beta_1 MKT + \beta_2 SMB + \beta_3 HML + \beta_4 MOM + \beta_5 LIQ + \varepsilon$

Regression:	**(18)**	**(19)**	**(20)**
Dependent Variable:	**Portfolio Excess Return Above 30-Day T-Bill (R_P - R_F)**		
	Top 25 Portfolios	**Top 50 Portfolios**	**Q1 Portfolios**
Factor (β)			
MKT (R_M - R_F)	1.66	1.67	1.63
	(7.69)	(7.58)	(6.62)
Intercept (CAPM α)	9.56%	7.49%	6.74%
	(2.32)	(1.79)	(1.44)
R^2	55.73%	54.98%	48.27%
Number of obs	49	49	49

Regression:	**(21)**	**(22)**	**(23)**
Dependent Variable:	**Portfolio Excess Return Above 30-Day T-Bill (R_P - R_F)**		
	Top 25 Portfolios	**Top 50 Portfolios**	**Q1 Portfolios**
Factors (β)			
MKT (R_M - R_F)	1.46	1.46	1.41
	(7.95)	(9.27)	(8.29)
SMB	1.09	1.13	1.09
	(3.94)	(4.72)	(4.24)
HML	0.55	0.66	0.77
	(2.25)	(3.17)	(3.42)
MOM	-0.77	-0.91	-1.11
	-(3.10)	-(4.26)	-(4.78)
LIQ	-0.26	-0.10	-0.34
	-(0.98)	-(0.45)	-(1.34)
Intercept (α)	**13.06%**	**10.92%**	**12.33%**
	(2.77)	**(2.69)**	**(2.81)**
R^2	78.88%	84.57%	83.36%
Number of obs	46	46	46

As shown in regressions 21 – 23, the portfolios of leveraged small-value stocks do not have a statistically significant liquidity beta. This is an important result given the fact that these portfolios were formed with a tilt towards stocks that had low share turnover in the preceding year. Therefore, Figure 10 provides evidence that the relatively low share turnover of these stocks is not due to illiquidity. Our hypothesis is that these stocks had low share turnover because there wasn't high investor sentiment for these companies so they didn't exhibit glamour characteristics.[10] This hypothesis is supported by the fact the stocks in these portfolios had below-median returns in the preceding year. As such, the companies in these portfolios exhibit the characteristics that we would expect from value stocks.

Most importantly, regressions 21 – 23 show that all three value-weighted portfolios have positive risk-adjusted returns that are statistically significant at the 5% level. The *Top 25 Portfolios* have a risk-adjusted average annual return of 13.1%. The risk-adjusted average annual returns of the *Top 50 Portfolios* and the *Q1 Portfolios* are 10.9% and 12.3%, respectively.

E. Comparison with Private Equity and Leveraged Indices

The success of the private equity industry provides additional empirical support for the ideas contained in this paper – though this paper argues that it is a combination of leverage and value in driving free cash flow yields that drives excess returns, not the private or public nature of the investment. The excess returns of private equity can be explained through the use of leverage to finance acquisitions of cheap companies. The key advantage of private ownership of leveraged businesses, however, is that the private equity investor can mask volatility because the equity securities are not publicly listed. With respect to value, the purchase multiple (*EV/EBITDA*) is one of the most important predictors of returns in private equity. It is therefore no surprise that

[10] "A Model of Investor Sentiment." Nicholas Barberis, Andrei Shleifer and Robert Vishny (1998)

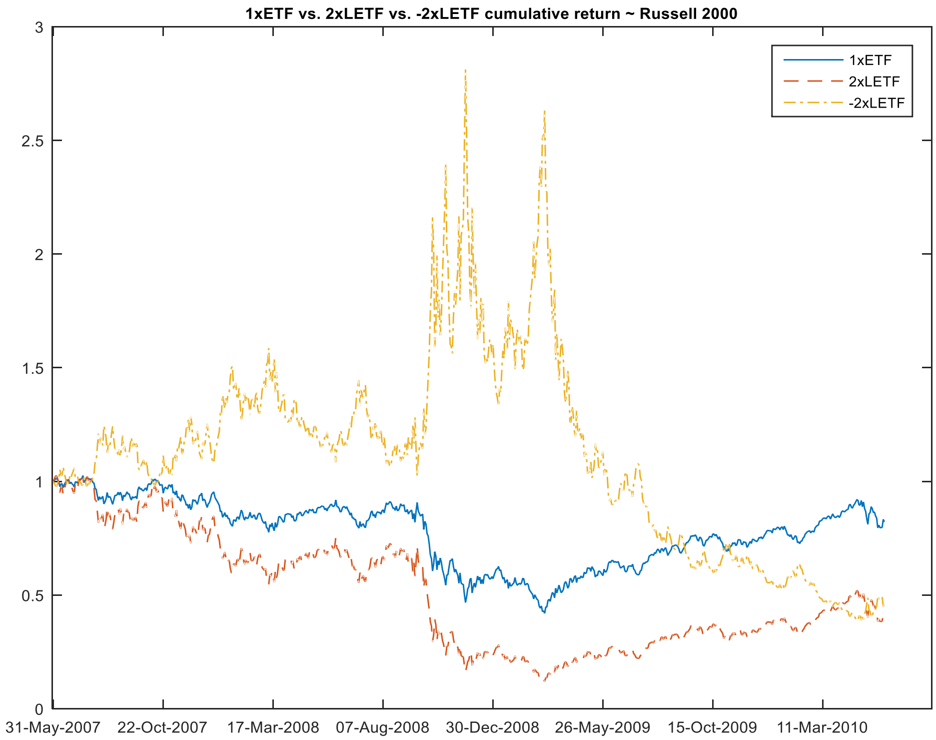
private equity returns are higher in vintage years that reflect lower purchase multiples. The relationship between purchase multiples and returns in private equity is related to the value effect in public markets, where low valuations (e.g. low *Price/Earnings* or low *Market/Book*) reflect higher expected returns. As shown in Figure 9, our strategy has a positive alpha relative to the Cambridge Associates Private Equity Index, although this result is not statistically significant.

Another alternative for accessing leveraged equity returns is to leverage broader market indices by buying leveraged ETFs (LETFs) that provide pre-packaged leverage relative to an index. However, these LETFs are designed to amplify short-term returns. LETFs rebalance daily in order to maintain a fixed leverage ratio relative to the underlying index, which increases their direct trading costs and raises their probability of underperformance relative to the index over the long term. Consider the example of a 2x LETF that has $2 million invested in an index, financed by $1 million in equity and $1 million of debt. If the underlying index goes up by 5% in a day, so that the 2x LETF's assets are worth $2,100,000 and its equity is worth $1,100,000, the 2x LETF is forced to borrow an additional $100,000 in order to buy more of the index and maintain 2x leverage. This occurs precisely when the index's stock prices go up. Similarly, if the underlying index goes down by 5% in a day, the 2x LETF is forced to sell $100,000 of stock that day at lower prices in order to return money to the lender and maintain 2x leverage. This feature of buying high and selling low generally lowers the appeal of LETFs as long term investments, especially during periods of higher volatility. As an example, Figure 11 presents the performance of an unlevered Russell 2000 ETF relative to a 2x LETF and a -2x LETF of the Russell 2000 Index between May 31, 2007 and May 31, 2010.[11] Both LETFs underperformed the unlevered ETF on a cumulative basis by the end of the time horizon, as a result of higher volatility during

[11] This LETF example is based on coursework in Professor Luboš Pástor's Portfolio Management Class at Chicago Booth. In Figure 11, we used actual returns of LETFs that are based on the Russell 2000 Index.

the financial crisis. While the Russell 2000 is not an ideal benchmark for a small-value strategy, we use it in this example because—unlike the CRSP Small Value Index—the Russell 2000 has LETFs that are actively traded.

Figure 11 – Unlevered (1x ETF) Relative to a 2x LETF and a -2x LETF on the Russell 2000



Source: CRSP data on daily ETF returns between May 31, 2007 and May 31, 2010

An LETF performs symmetrically to the underlying index and it has questionable value as a long term investment. Conversely, portfolios of leveraged small-value stocks are arguably more likely to provide a positive alpha relative to their index over the long term, as suggested by the results in Figure 9.

6. Discussion and Conclusions

The value investors who loom large in the popular imagination eschew leverage. Benjamin Graham once described corporate debt as an "adverse economic factor of some magnitude and a real problem for many individual enterprises"[12] Warren Buffett later said, "I do not like debt and do not like to invest in companies that have too much debt, particularly long-term debt."[13] Avoiding highly leveraged firms has become conventional wisdom in the value investing community.

Yet leveraged buyout firms have realized for years that increasing leverage can juice returns – and the spectacular track record of the private equity industry and the clamor of pensions and endowments to invest further in private equity funds speaks to the attractive returns generated by buying companies with borrowed money.

We propose that value investors have something to learn from the Barbarians at the Gates – and our research points to the ways in which leverage enhances a small-value strategy. Investors have a well-documented pattern of leverage aversion, which we believe contributes to the excess returns to be found in leveraged small-value stocks.[14] Our research goes beyond simply identifying leverage as an enhancement to small-value strategies by offering a method for distinguishing the attractive leveraged investments from the unattractive ones. We consider leverage, value, and size in combination with quality factors that serve to reduce risk. We argue that it is critical for investors to select firms where management is already making responsible capital allocation decisions, such as paying down debt.

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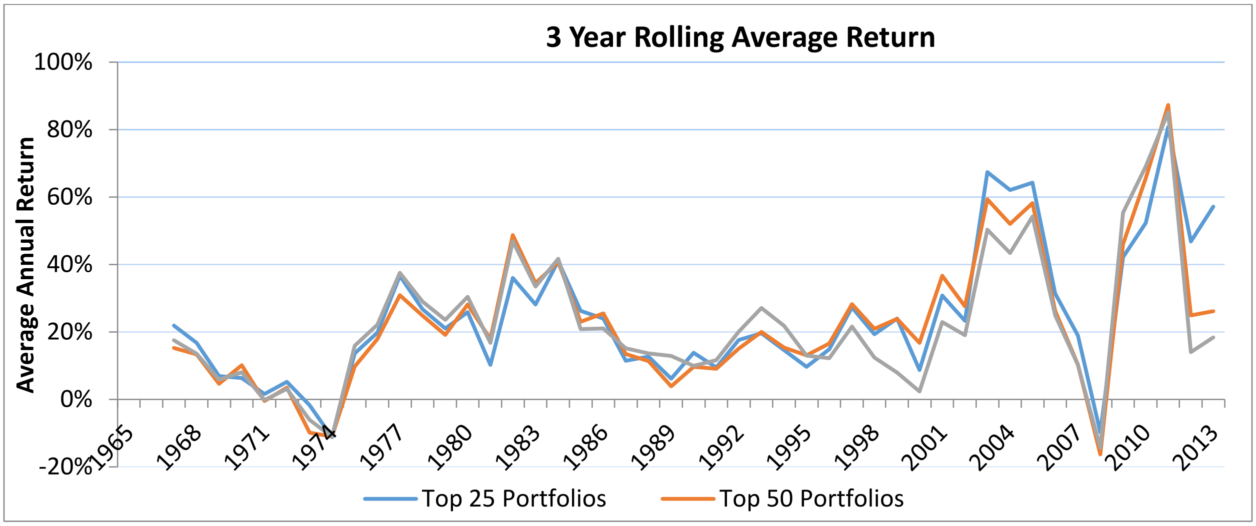
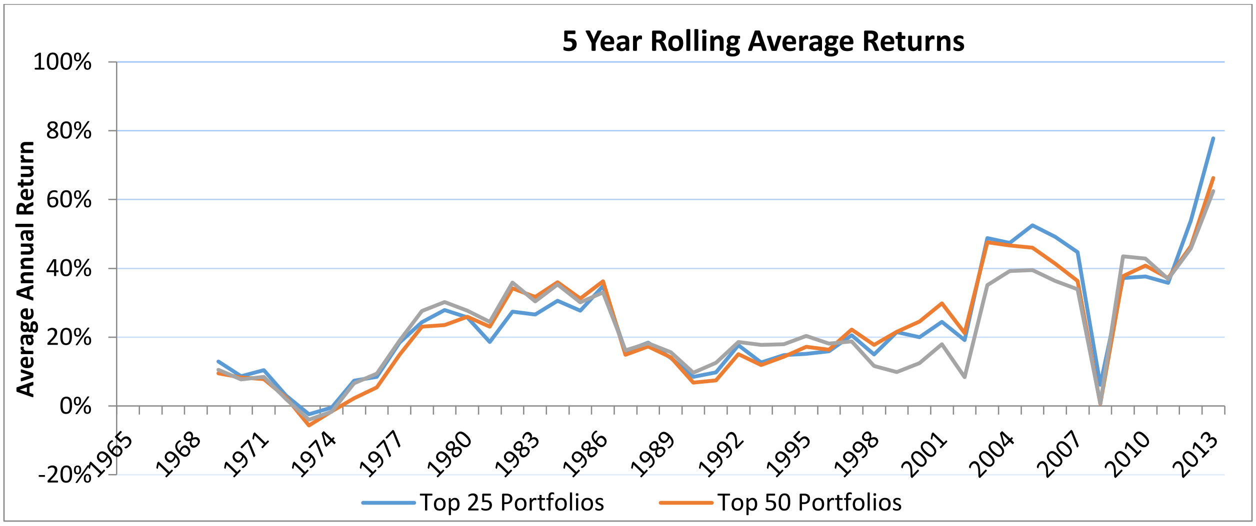
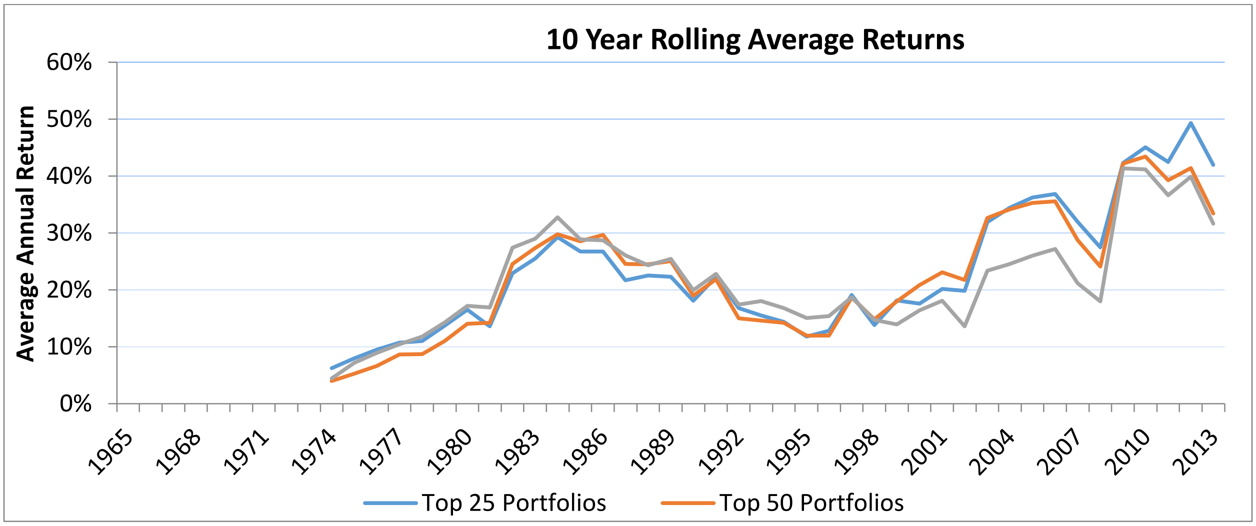
[12] Benjamin Graham, *The Intelligent Investor*, New York, 1954.
[13] http://www.minterest.org/best-warren-buffett-quotes-on-investing
[14] "Leverage Aversion and Risk Parity" Clifford S. Asness, Andrea Frazzini, and Lasse H. Pedersen (2012)

The greatest challenge to this strategy is the volatility of returns, which is significantly higher than broader market indices. Private equity has solved this problem by taking the companies private and thus masking the price volatility. But it is unavoidable in the public markets and can only be solved with a long-term and disciplined approach. Specifically, it is essential to stick with a systematic investment approach over time in order to reap the full benefits of this strategy, as demonstrated by the rolling average value-weighted returns in Figure 12 below.

Figure 12 – Rolling Value-Weighted Avg. Returns of Leveraged Small-Value Portfolios







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